

# A guide to exchange-correlation functionals

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**CASTEP Training Workshop 2019** 

# Exchange-correlation functionals: A zoo



	functional	like		neutral		hate		blanc		none		#votes		points	average
1	PBE	26	55.3 %	9	19.1 %	3	6.4 %	3	6.4 %	6	12.8 %	41	87.2 %	84	1.79
2	PBE0	25	53.2 %	8	17.0 %	4	8.5 %	4	8.5 %	6	12.8 %	41	87.2 %	79	1.68
3	PBE-D	21	44.7 %	7	14.9 %	5	10.6 %	6	12.8 %	8	17.0 %	39	83.0 %	65	1.38
4	wB97X-D	19	40.4 %	7	14.9 %	4	8.5 %	13	27.7 %	4	8.5 %	43	91.5 %	60	1.28
5	B3LYP	17	36.2 %	9	19.1 %	14	29.8 %	4	8.5 %	3	6.4 %	44	93.6 %	46	0.98
6	CAM-B3LYP	14	29.8 %	9	19.1 %	6	12.8 %	12	25.5 %	6	12.8 %	41	87.2 %	45	0.96
7	B3LYP-D	13	27.7 %	9	19.1 %	9	19.1 %	10	21.3 %	6	12.8 %	41	87.2 %	39	0.83
8	B2PLYP	11	23.4 %	11	23.4 %	6	12.8 %	12	25.5 %	7	14.9 %	40	85.1 %	38	0.81
9	wB97X-V	11	23.4 %	6	12.8 %	3	6.4 %	18	38.3 %	9	19.1 %	38	80.9 %	36	0.77
10	HSE	9	19.1 %	14	29.8 %	5	10.6 %	11	23.4 %	8	17.0 %	39	83.0 %	36	0.77
11	PW91	9	19.1 %	15	31.9 %	7	14.9 %	8	17.0 %	8	17.0 %	39	83.0 %	35	0.74
12	B97-D	10	21.3 %	8	17.0 %	4	8.5 %	18	38.3 %	7	14.9 %	40	85.1 %	34	0.72
13	M06-2X	13	27.7 %	5	10.6 %	13	27.7 %	9	19.1 %	7	14.9 %	40	85.1 %	31	0.66
14	revPBE	8	17.0 %	10	21.3 %	5	10.6 %	14	29.8 %	10	21.3 %	37	78.7 %	29	0.62
15	TPSSh	8	17.0 %	9	19.1 %	5	10.6 %	16	34.0 %	9	19.1 %	38	80.9 %	28	0.60
16	PWPB95-D3	8	17.0 %	7	14.9 %	5	10.6 %	18	38.3 %	9	19.1 %	38	80.9 %	26	0.55
17	wB97M-V	7	14.9 %	8	17.0 %	3	6.4 %	18	38.3 %	11	23.4 %	36	76.6 %	26	0.55
18	LC-wPBE	7	14.9 %	9	19.1 %	5	10.6 %	16	34.0 %	10	21.3 %	37	78.7 %	25	0.53
19	BP86	6	12.8 %	9	19.1 %	8	17.0 %	14	29.8 %	10	21.3 %	37	78.7 %	19	0.40
20	B97M-V	4	8.5 %	9	19.1 %	4	8.5 %	21	44.7 %	9	19.1 %	38	80.9 %	17	0.36

Table 1. The L1 methods (yielding the best							
densities), their rungs, years, and normal-							
ized errors (NE).							

Method	Rung	Year	Max NE
CCSD	Ab initio		0.000
MP4sdq	Ab initio		0.246
MP3	Ab initio		0.967
MP2	Ab initio		1.514
mPW3PBE	hGGA	1998	1.778
APFD	hGGA	2012	1.813
B3PW91	hGGA	1993	1.816
PBEO	hGGA	1999	1.818
B98	hGGA	1998	1.826
BHHLYP	hGGA	1993	1.851
B97-3	hGGA	2005	1.883
mPW1PBE	hGGA	1998	1.910
B3P86	hGGA	1993	1.937
03LYP	hGGA	2001	1.947
PBE1KCIS	hGGA	2005	1.954
mPW1PW91	hGGA	1998	1.955
B97-1	hGGA	1998	1.962
HSE06	hGGA	2006	1.982
PBEh1PBE	hGGA	1998	1.983
B97-2	hGGA	2001	2.018
B1B95	hGGA	1996	2.033
TPSS	mGGA	2003	2.042
TPSSh	hGGA	2003	2.045
TPSSm	mGGA	2007	2.077
X3LYP	hGGA	2005	2.084
SCAN	mGGA	2015	2.107
B3LYP	hGGA	1993	2.123

Table 2. The L2 methods (yielding the worst densities), their rungs, years, and normalized errors (NE).

Method	Rung	Year	Max NE
Χα*	LDA	1974	3.777
SP86	GGA	1986	3.821
M06-L	mGGA	2006	3.974
SVWN1RPA	LDA	1980	3.977
SPBE	GGA	1997	3.978
SVWN	LDA	1980	3.984
SPZ81	LDA	1981	3.985
SPW91	GGA	1991	3.989
M06-2X	hGGA	2006	4.027
SOP	GGA	1997	4.182
SLYP	GGA	1988	4.429
M08-S0	hGGA	2008	4.676
SLATER*	LDA	1974	4.864
M08-HX	hGGA	2008	4.880
SOGGA11	GGA	2011	4.971
M06	hGGA	2006	5.420
M06-HF	hGGA	2006	6.125
N12	GGA	2012	6.709
N12-SX	hGGA	2012	6.970
M05	hGGA	2005	7.652
MN12-L	mGGA	2012	8.995
M11	hGGA	2011	10.191
MN12-SX	hGGA	2012	13.005
M11-L	mGGA	2011	15.316

 $\alpha^* = 0.7$  for X $\alpha$  and  $\alpha = 2/3$  for SLATER (see supplementary materials for references).

Ab initio?



#### **Density Functional Theory**

Many-body Schrödinger equation (the "truth"):  $\hat{H}\Psi(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N) = E\Psi(\mathbf{r}_1,\mathbf{r}_2,\ldots,\mathbf{r}_N)$ Hohenberg-Kohn theorems Problem to a one-body problem  $\Psi(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N) \rightarrow n(\mathbf{r})$  $E[n] = F[n] + \left[ d\mathbf{r} \ V(\mathbf{r})n(\mathbf{r}) = T[n] + U[n] + \left| d\mathbf{r} \ V_{\text{ext}}(\mathbf{r})n(\mathbf{r}) \right. \right]$ Kohn-Sham formulation  $E[n] = -\frac{1}{2} \sum_{i=1}^{N} \nabla^2 \psi_i + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + E_{xc}[n] + \int d\mathbf{r} \ V_{ext}(\mathbf{r})n(\mathbf{r}) \qquad n(\mathbf{r}) = \sum_{i=1}^{N} |\psi_i(\mathbf{r})|^2$ Science & Technology

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#### **Density Functional Theory**

Kohn-Sham formulation

$$E[n] = -\frac{1}{2} \sum_{i=1}^{N} \nabla^2 \psi_i + \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} + \int d\mathbf{r} \ V_{\text{ext}}(\mathbf{r})n(\mathbf{r}) + E_{\text{xc}}[n] \quad n(\mathbf{r}) = \sum_{i=1}^{N} |\psi_i(\mathbf{r})|^2$$

Kinetic energy of the non-interacting particles

$$T_{s}[n] = -\frac{1}{2} \sum_{i=1}^{N} \nabla^{2} \psi_{i}$$
 vs.  $T[n]$ 

Hartree-term: Coulombic interaction of electrons

 $E_{H}[n] = \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \quad \text{vs.} \quad U[n]$ 

Exchange-correlation functional has to model:

- kinetic energy of the many-body system
- non-Coulombic interactions

$$E_{\rm xc}[n] = \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \, n(\mathbf{r}) \frac{n(\mathbf{r}, \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$



#### **Exchange-correlation functionals**

- The form of the exchange-correlation functional is unknown
- There exist a series of approximations: "Jacob's ladder" to chemical accuracy
- Chemical accuracy: thermochemical energetics (1 kcal/mol error in formation energies), molecular geometries
- Errors in DFT can be attributed to the limitation of XC functionals
  - Self-interaction error (electron selfrepulsion)
  - Derivative discontinuity
  - Non-local correlation weak interactions



from N Mardirossian and M Head Gordon, Mol. Phys. **115** 2315

$$E_{H}[n] = \frac{1}{2} \int d\mathbf{r} d\mathbf{r}' \frac{n(\mathbf{r})n(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

 $E(N+\delta) = (1-\delta)E(N) + \delta E(N+1)$ 



#### **Exchange-correlation functionals**



from N Mardirossian and M Head Gordon, Mol. Phys. **115** 2315

 $\varepsilon_{\text{LDA}}, \varepsilon_{\text{GGA}}$  and  $\varepsilon_{\text{mGGA}}$  are *unknown*, need fitting!



#### Generalities

- All XC functionals are approximations
  - DFT (the theory) is exact
  - Reports of 'Failures of DFT' are actually reports of the failure of the XC functional (approximation)
- No functional (so far) is accurate for all properties of interest
  - No matter what functional is invented, someone will always find a case where it fails
- Any functional can be applied to any electronic structure problem
  - But they have known limitations

 It is still ab initio, but we use experience, benchmarks and intuition to decide which ones to use



#### **Local Density Approximation**

$$E_{\rm xc} = \int d\mathbf{r} \ \varepsilon_{\rm LDA}(n(\mathbf{r}))n(\mathbf{r})$$

Typical errors:

- Binding energies too large
- Bonds and lattice parameters too small
- Incorrect order of phase stability
- Error in energetics in magnetic materials

Curtiss et al.: Computation of enthalpies of formation

1073

TABLE IV. Deviation	of enthalpies	calculated by	DFT	methods	with	experiment. <sup>a</sup>
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		Deviation (ExptTheory)							
Molecule	LDA	BLYP	BP86	BPW91	B3LYP	B3P86	B3PW91		
G2-1 test set									
LiH	3.3	0.1	0.5	-4.6	0.4	1.2	-3.3		
BeH	11.2	7.3	8.5	6.5	8.2	10.0	7.4		
СН	9.9	1.8	4.1	-0.1	1.7	3.9	0.0		
$CH_2({}^{3}B_1)$	24.1	0.0	8.5	2.8	2.1	10.3	4.2		
$CH_2(^1A_1)$	20.7	-0.6	4.2	-3.7	0.2	4.9	-2.7		
CH <sub>3</sub>	35.2	0.4	10.4	0.3	3.3	13.1	2.9		
CH <sub>4</sub>	46.8	-2.3	10.4	-3.4	1.6	14.1	0.2		
NH	13.7	6.0	8.6	3.8	4.6	7.1	2.5		
NH <sub>2</sub>	29.7	8.0	13.5	4.3	6.5	11.9	3.0		
NH <sub>3</sub>	44.3	4.3	13.3	0.3	3.5	12.4	-0.4		
OH	19.5	3.4	6.6	2.0	1.8	5.1	0.4		
OH <sub>2</sub>	37.3	0.6	7.8	-0.9	-1.3	5.8	-3.1		
FH	22.9	0.4	4.1	-0.2	-1.6	2.2	-2.3		

L. A. Curtiss *et al*, *JCP* **106**, 1063



#### **LDA** parameterisations

$$E_{\rm xc} = \int d\mathbf{r} \ \varepsilon_{\rm LDA}(n(\mathbf{r}))n(\mathbf{r})$$

XC\_FUNCTIONAL : LDA

- Exchange: Slater exchange
- Correlation: parameterisations based on accurate calculations of the Homogenous Electron Gas
  - Vosko-Wilk-Nusair 5 (VWN5): Can. J. Phys. 58, 1200 (1980)
  - Perdew-Zunger: PRB 23, 5048 (1981)
  - Perdew-Wang: PRB 45, 13244 (1992)
  - VWN3: Can. J. Phys. 58, 1200 (1980)

 $\varepsilon_{\rm LDA,x} \propto n^{1/3}$ 



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#### **Generalised Gradient Approximations**

$$E_{\rm xc} = \int d\mathbf{r} \ \varepsilon_{\rm GGA}(n(\mathbf{r}), \nabla n(\mathbf{r}))n(\mathbf{r})$$

- More flexible functional form
- Semi-local: information about nearby regions via the gradient
- Not very successful before 1996 (PBE: Generalised Gradient Approximation Made Simple)
- Theoretical limits and constraints explicitly built in
- Modifies LDA exchange and correlations using "enhancement factors"



## **Generalised Gradient Approximations**

	Latti	ce const	ants (Å)		Binding energies (eV / atom)					
Solids	LSDA	PBE	PBEsol	Expt.	LSDA	PBE	PBEsol	Expt.		
Li	3.362	3.431	3.426	3.453	1.810	1.605	1.677	1.658		
Na	4.051	4.198	4.170	4.214	1.256	1.082	1.154	1.119		
Ca	5.332	5.518	5.448	5.553	2.220	1.917	2.117	1.86		
Sr	5.791	6.027	5.916	6.045	1.893	1.609	1.808	1.73		
Ba	4.770	5.030	4.894	4.995	2.246	1.871	2.109	1.91		
Al	3.983	4.035	4.011	4.018	4.038	3.438	3.817	3.431		
Cu	3.522	3.633	3.565	3.595	4.545	3.474	4.027	3.524		
Rh	3.759	3.831	3.781	3.794	7.563	5.688	6.642	5.783		
Pd	3.844	3.942	3.876	3.876	5.016	3.714	4.435	3.938		
Ag	4.002	4.145	4.050	4.062	3.642	2.516	3.078	2.985		
C	3.533	3.569	3.552	3.553	9.011	7.714	8.275	7.545		
SiC	4.332	4.378	4.355	4.346	7.457	6.401	6.876	6.478		
Si	5.405	5.468	5.432	5.421	5.348	4.559	4.940	4.685		
Ge	5.631	5.768	5.680	5.644	4.628	3.716	4.144	3.918		
GaAs	5.615	5.752	5.665	5.640	4.095	3.148	3.555	3.337		
LiF	3.913	4.070	4.007	3.972	4.945	4.322	4.474	4.457		
LiCl	4.968	5.151	5.063	5.070	3.835	3.364	3.518	3.586		
NaF	4.506	4.705	4.632	4.582	4.384	3.826	3.959	3.970		
NaCl	5.467	5.695	5.603	5.569	3.503	3.097	3.223	3.337		
MgO	4.170	4.261	4.223	4.189	5.863	4.973	5.299	5.203		
ME	-0.082	0.051	-0.012		0.642	-0.121	0.234			
MAE	0.082	0.060	0.035		0.642	0.144	0.253			
MRE(%)	-1.73	1.10	-0.24		16.50	-3.68	5.97			
MARE(%)	1.73	1.29	0.73		16.50	4.23	6.52			

J. Sun et al, PRB 84, 035117



#### **GGA** parameterisations

$$E_{\rm xc} = \int d\mathbf{r} \ \varepsilon_{\rm GGA}(n(\mathbf{r}), \nabla n(\mathbf{r}))n(\mathbf{r})$$

XC\_FUNCTIONAL : PW91
J. P. Perdew and Y. Wang, PRB 45 13244 (1992)

XC\_FUNCTIONAL : PBE

 J. P. Perdew, K. Burke and M Ernzerhof, *PRL* 77 3865 (1996)

XC FUNCTIONAL : RPBE

 B. Hammer, L. B. Hansen and J. K. Norskov, *PRB* 59 7413 (1999)

XC\_FUNCTIONAL : WC

• Z. Wu and R. E. Cohen, PRB 73, 235116 (2006)



#### **Meta-Generalised Gradient Approximations**

$$E_{\rm xc} = \int d\mathbf{r} \ \varepsilon_{\rm mGGA}(n(\mathbf{r}), \nabla n(\mathbf{r}), \nabla^2 n(\mathbf{r}), \tau(\mathbf{r}))n(\mathbf{r})$$
  
$$\tau(\mathbf{r}) = \frac{1}{2} \sum_{i=1}^{N} |\nabla \psi(\mathbf{r})|^2 \qquad V_{\rm xc}(\mathbf{r}) = \frac{\delta E_{\rm xc}}{\delta n} = V_{\rm GGA}(\mathbf{r}) + \hat{V}_{\tau}(\mathbf{r})$$

- Even more flexible functional form
- Still semi-local
- Not very successful before 2015 (SCAN: Strongly Constrained and Appropriately Normed functional)
- More theoretical limits, constraints and accurate cases explicitly built in
- Kinetic energy density: solve the Generalised Kohn-Sham equations (XC potential not multiplicative)
- Improved energetics and structural properties
- Band gaps closer to experimental ones



#### mGGA parameterisations

$$E_{\rm xc} = \int d\mathbf{r} \ \varepsilon_{\rm mGGA}(n(\mathbf{r}), \nabla n(\mathbf{r}), \nabla^2 n(\mathbf{r}), \tau(\mathbf{r}))n(\mathbf{r}) \qquad \tau(\mathbf{r}) = \frac{1}{2} \sum_{i=1}^{N} |\nabla \psi(\mathbf{r})|^2$$

$$V_{\rm mBJ}(\mathbf{r}) = V(n(\mathbf{r}), |\nabla n(\mathbf{r})|, \tau(\mathbf{r})) \qquad V_{\rm mGGA}(\mathbf{r}) = \frac{\delta E_{\rm xc}}{\delta n} = V_{\rm GGA}(\mathbf{r}) + \hat{V}_{\tau}(\mathbf{r})$$
PRL 102, 226401 (2009)
- potential only mGGA, no total energy, no forces etc.
- improves band gaps
• TPSS mGGA: Tao, Perdew, Staroverov, Scuseria,
PRL 91, 146401 (2003)
$$XC_{\rm FUNCTIONAL} : \mathbf{r}SCAN$$
• J. Sun, A. Ruzsinszky, J. P. Perdew
PRL 115 036402 (2015)

ШШ

0.0

12

14

16

18

Volume (Å<sup>3</sup> per atom)

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22

20

Nat. Chem. 8, 831

- functional form obeys limits by construction
- bonding situation determined based on KED
- limiting cases are interpolated

#### **Hybrid functionals**

$$E_{\text{x,exact}} = -\frac{1}{2} \sum_{i,j} \int d\mathbf{r} d\mathbf{r}' \frac{\psi_i^*(\mathbf{r})\psi_i(\mathbf{r}')\psi_j^*(\mathbf{r})\psi_j(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$
$$E_{\text{x}} = (1 - \alpha)E_{\text{x,GGA}} + \alpha E_{\text{x,exact}}$$
$$V_{\text{x,exact}}(\mathbf{r}, \mathbf{r}') = -\frac{1}{2} \sum_j \frac{\psi_j^*(\mathbf{r})\psi_j(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|}$$

- Exact exchange: corresponds to the exchange energy of non-interacting one-body orbitals - does not account for all exchange energy
- Hybrid functionals use a mix of GGA exchange and exact exchange, as well as GGA correlation
- Non-local potential: a lot more expensive in planewaves than semi-local functionals



#### Hybrid functional parameterisations

In CASTEP:

- Worse scaling with system size more expensive
- Not all functionality present

```
XC_FUNCTIONAL : B3LYP
```

 P. J. Stephens, F. J. Devlin, C. F. Chabalowski and M. J. Frisch, JPC 98 11623 (1994)

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XC_FUNCTIONAL : PBE0
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• C. Adamo and V. Barone, JCP 110 6158 (1999)

XC\_FUNCTIONAL : HSE03

• J. Heyd, G. E. Scuseria and M. Ernzerhof, JCP 118 8207 (2003)

XC\_FUNCTIONAL : HSE06

 A. V. Krukau, O. A. Vydrov, A. F. Izmaylov, and G. E. Scuseria, *JCP* 125 224106 (2006)



range-separated hybrids (more efficient for metals)

#### **Double hybrid functionals**

- Hybrid exchange and
- In a similar spirit to hybrids, correlation is a mix between GGA correlation and MP2 correlation
- Weak interactions (van der Waals) better described
- Not available in CASTEP



Acc. Chem. Res. 49, 1503



## DFT+U



- On-site repulsion not treated properly
- Correlation effects not modelled accurately especially in open-shell systems
- Electrons in LDA and GGA tend to be more delocalised than they should be
- meta-GGAs (SCAN) appears to improve the description of strongly correlated systems *without* U

%block hubbard\_u
Ni d:5.0
%endblock hubbard\_u





PRB **73,** 235206



#### **Dispersion corrections**

- Not all correlation effects accounted for in DFT (particularly non-local correlation)
- van der Waals interactions significant in a number of systems:
  - Molecular crystals
  - Layered materials
  - Even oxides!
- Dispersion correction:
  - Pairwise schemes
  - Three-body interactions
  - TS pairwise parameters depend on electronic structure
  - many-body dispersion



JCP 140, 18A508 sedc apply : true

sedc scheme : OBS

• PRB 73, 205101 (2006)

#### sedc\_scheme : G06

• (D2) J. Comput. Chem. 27, 1787 (2006)

#### sedc\_scheme : JCHS

- J. Comput. Chem. 28, 555 (2007)
- sedc\_scheme : TS
- PRL 102, 073005 (2009)

#### sedc\_scheme : MBD\*

• J. Chem. Phys. 140, 18A508 (2014)



#### **Dispersion corrections**



$\Delta E_{\mathrm{I} \rightarrow \mathrm{II}}/\mathrm{kJ/mol}$
-0.18
-0.42
-0.68
0.04
0.35
2.56
ne
+MBD@rsSCS

**I**NG

0.17

Article. Published on 30 March 2015. Downloaded on 8/21/2019

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#### PRL 113, 055701







Chem. Sci., 6, 3289



# Which functional to use?

- LDA
  - nice covalent systems
  - simple metals
- DFT+U
  - strongly correlated systems (localised *d* and *f* orbitals)
  - Mott insulators
- GGA
  - molecules
  - hydrogen-bonded materials
  - transition metals
  - magnetic systems
- Hybrid and screened hybrid exchange
  - band gaps
  - magnetic systems
- Dispersion corrected DFT (mostly GGA)
  - molecular crystals
  - layered materials
- meta-GGA
  - careful optimism: improves on most properties, including mid-range van der Waals



